PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 05

By- Akhil Roy Sir

Today's Targets

1. Function
2. Domain
3. Range
4. Co-Domain

Homework

Let X = set of real numbers & $R = \{(x, y) : x - y + \sqrt{3}$ is irrational$\}$ be a relation on X then R is

A. Reflexive and Symmetric

B. Reflexive only ✓

C. Symmetric only

D. None of these

① Ref. ✓

$x - x + \sqrt{3}$

$= \sqrt{3} \in$ Irr.

$\Rightarrow (x, x) \in R$

② Symm. ✗

let $x = 2\sqrt{3}, y = \sqrt{3}$

$x - y + \sqrt{3} = 2\sqrt{3} - \sqrt{3} + \sqrt{3}$

$= 2\sqrt{3} \in$ Irr.

But,

$y - x + \sqrt{3} = \sqrt{3} - 2\sqrt{3} + \sqrt{3}$

$= 0 \notin$ Irr.

for $(2\sqrt{3}, \sqrt{3}) \in R$

but $(\sqrt{3}, 2\sqrt{3}) \notin R$

③ Trans. ✗

let $x = \sqrt{3}, y = 4\sqrt{3}, z = 2\sqrt{3}$

$x - y + \sqrt{3} = -2\sqrt{3} \in$ Irr.

$y - z + \sqrt{3} = 3\sqrt{3} \in$ Irr.

But $x - z + \sqrt{3} = 0 \notin$ Irr.

$(\sqrt{3}, 4\sqrt{3}), (4\sqrt{3}, 2\sqrt{3}) \in R$

But $(\sqrt{3}, 2\sqrt{3}) \notin R$

Homework

Let S be the set of all real numbers. Then the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is:

A. Reflexive and symmetric but not transitive

B. Reflexive, Symmetric and Transitive

C. Reflexive and transitive but not symmetric

D. None of these

① R ✓

$1 + a.a = 1 + a^2 > 0$

(+ve + +ve)

② S ✓

$1 + ab > 0$

$1 + ba > 0$

③ Trans. ✗

let $a = 2$ $b = \frac{1}{4}$ $c = -2$

✓ $1 + ab = 1 + \frac{1}{2} = \frac{3}{2} > 0$

✓ $1 + bc = 1 - \frac{1}{2} = \frac{1}{2} > 0$

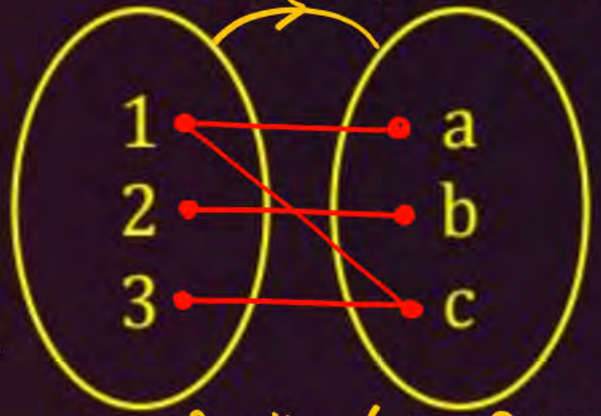
$1 + ac = 1 - 4 = -3 < 0$

Function (Class XI)

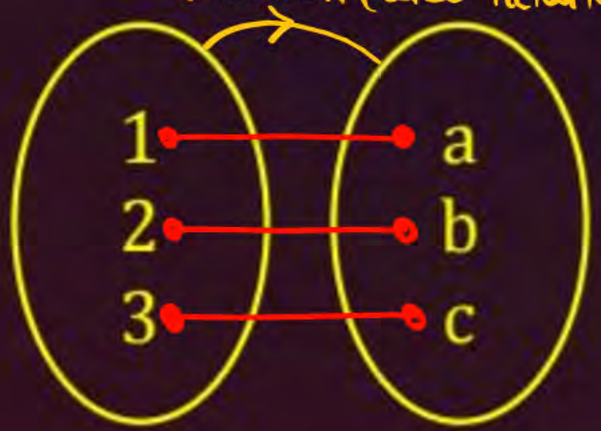
Every element in domain must have **one** and **only one image** in co-domain

* Every func. is relation but every relation is not function.

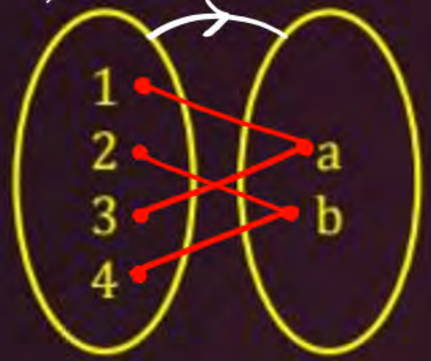
Relation (Not a function)



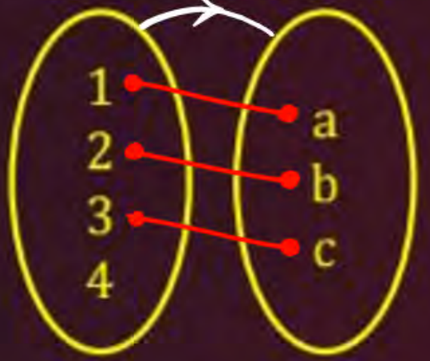
function (also Relation)



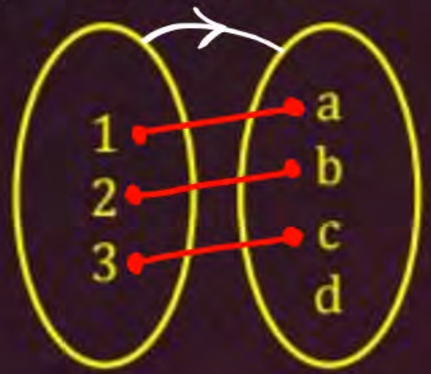
A) function



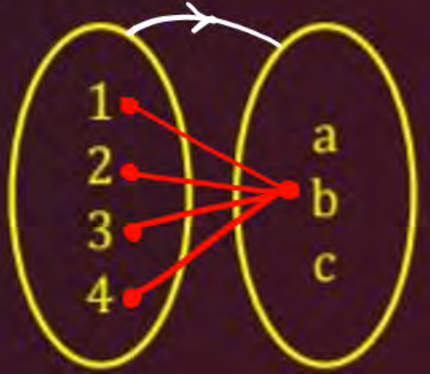
B) Not function → 2nd set



C) function



D) function

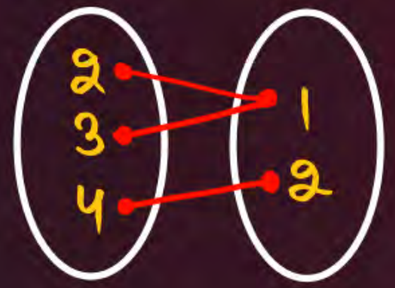


Question

Examine each of the following relations given below whether it is a function or not?

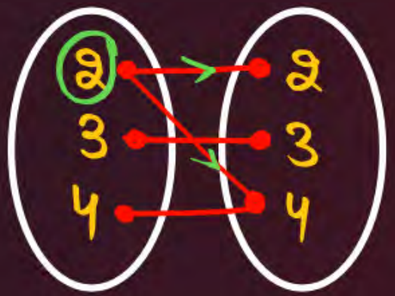
1) $R = \{(2,1), (3,1), (4,2)\}$ ⟶ It is a function
2) $R = \{(2,2), (2,4), (3,3), (4,4)\}$ ⟶ Not function
3) $R = \{(1,2), (2,3), (3,4), (4,5), (5,6), (6,7)\}$ → function

1)



3)

2)



Question

$f : Z \to Z$

Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

Sol: let $a = 2, b = 2$, let $a = -2, b = -2$

$\hookrightarrow (4, 4) \in f$ $\hookrightarrow (4, -4) \in f$

4 has two distinct images.

$\therefore$ f is not a function

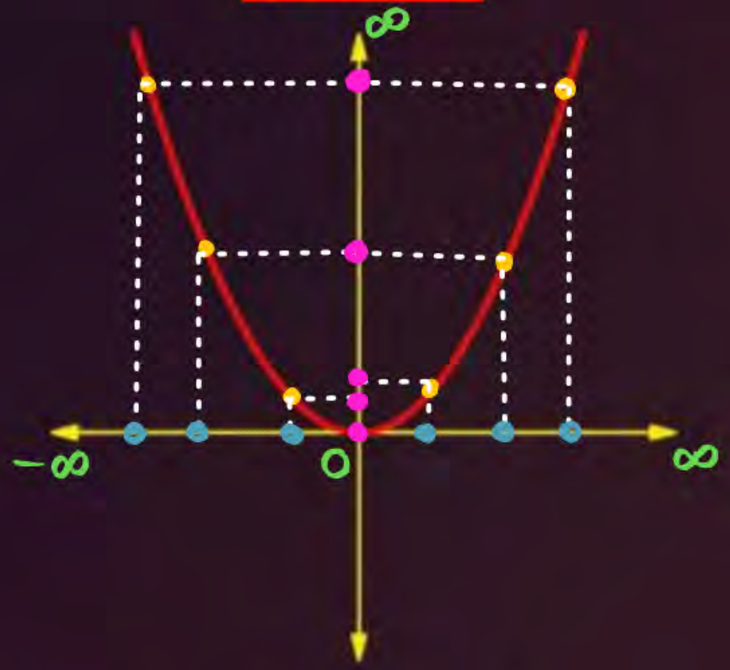
Graphs of Functions

$x \in$ Domain
$y \in$ Range

$f(x) = x$

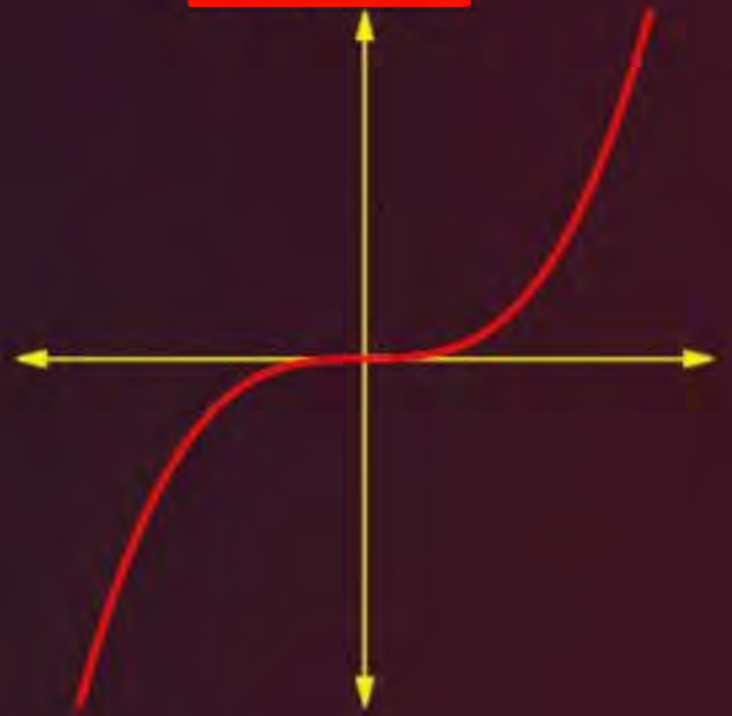
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$

$f(x) = x^2$



Domain $= (-\infty, \infty)$
Range $= [0, \infty)$

$f(x) = x^3$



Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$

Brackets

Open bracket ( ) → end points excluded

Closed bracket [ ] → end points included

① $0 \leq x \leq 2 \rightarrow [0,2] \rightarrow$

② $0 < x \leq 2 \rightarrow (0,2] \rightarrow$

③ $0 \leq x < 2 \rightarrow [0,2) \rightarrow$

④ $0 < x < 2 \rightarrow (0,2) \rightarrow$

① Google

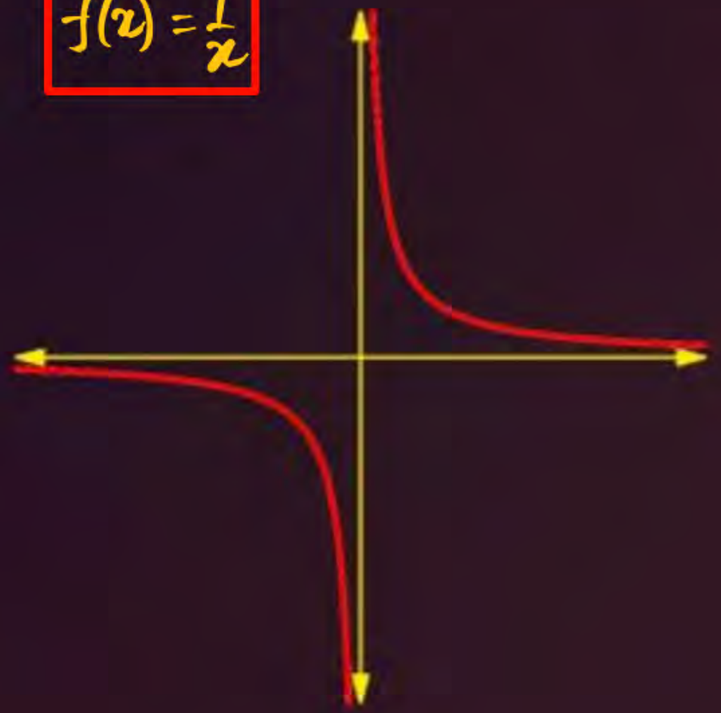
② Desmos Graphing Calculator

③ HW: Draw any graph you want to

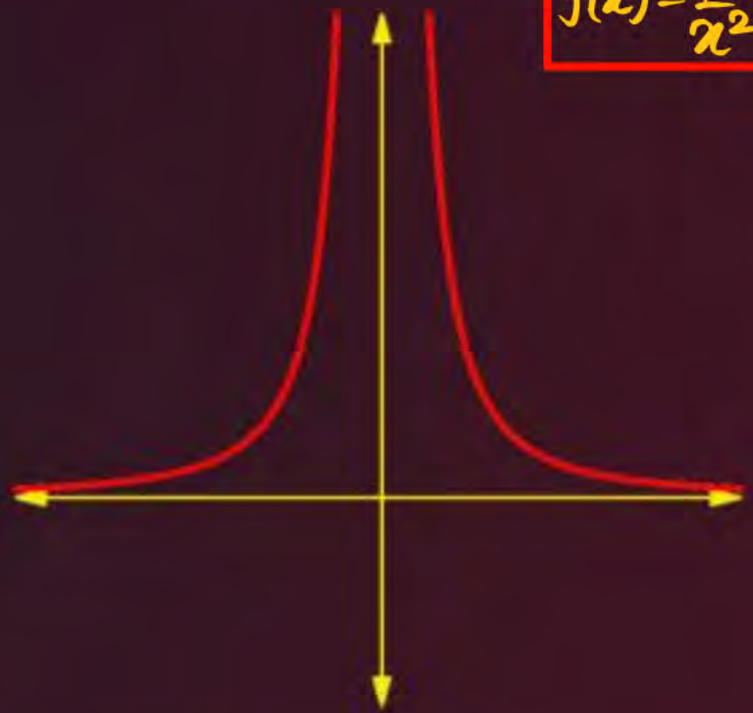
Graphs of Functions

$f(x) = |x|$

$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$

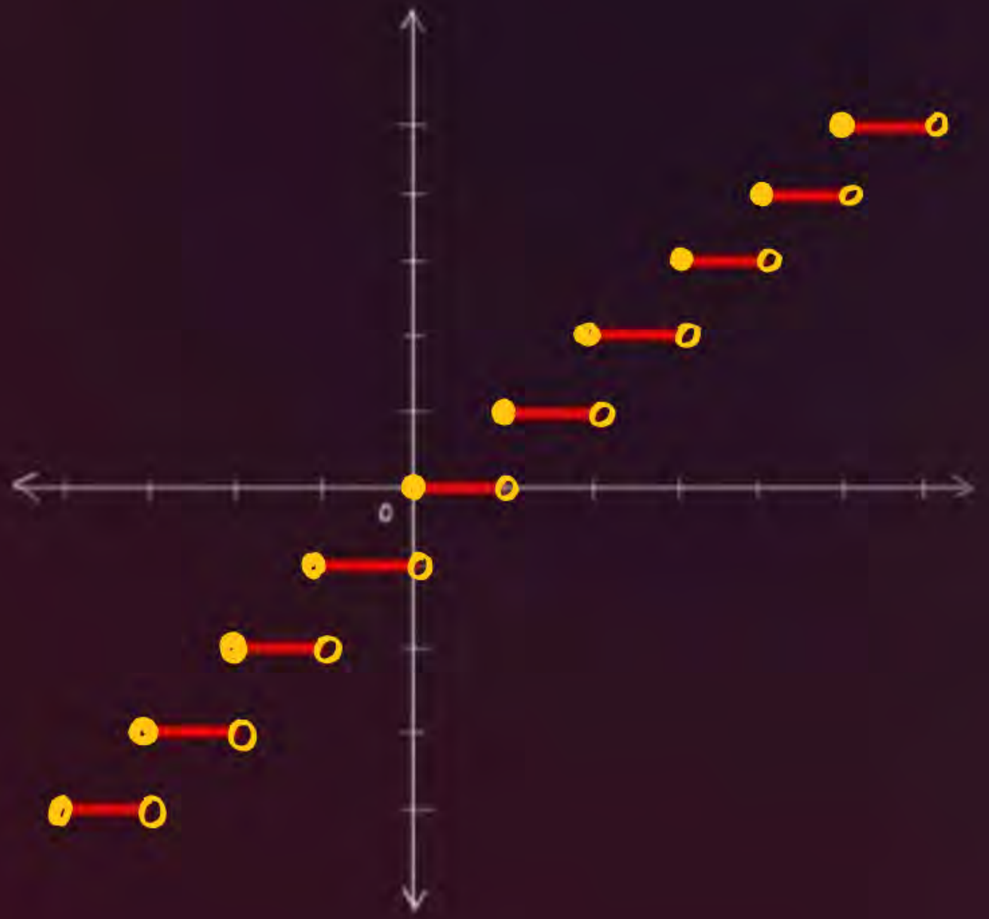


Graphs of Functions

$$f(x) = \text{sgn}(x) = \begin{cases} -1 & , \ x<0 \\ 0 & , \ x=0 \\ 1 & , \ x>0 \end{cases}$$

↳ It gives the sign of No.

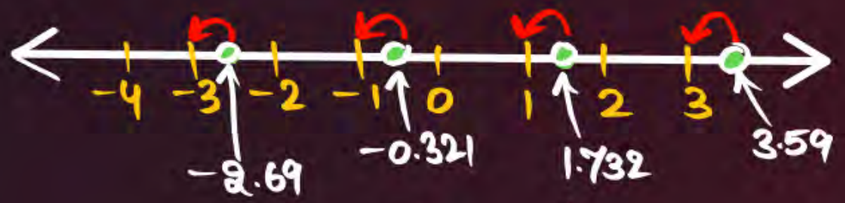
Graphs of Functions



$f(x) = [x]$

Challenge.

Domain = ?
Range = ?



$[3.59] = 3$ | $[3] = 3$

$[1.732] = 1$ | $[1] = 1$

$[-0.321] = -1$ | $[-1] = -1$

$[-2.69] = -3$ | $[-2] = -2$

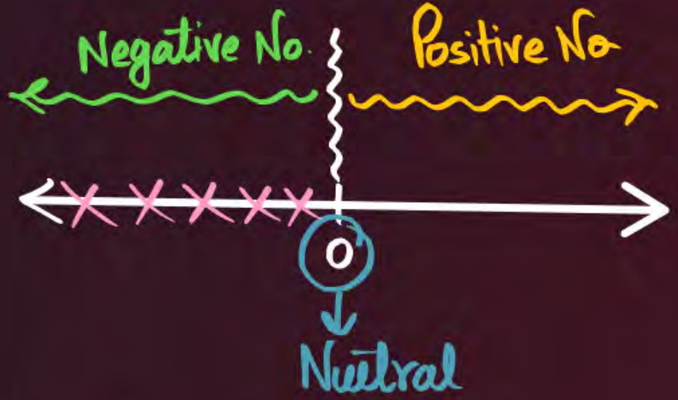
Domain, Range and Co-Domain

2nd set

① **Denomiator ≠ 0**

$\frac{1}{0}$ = Not defined

② $\sqrt{\text{non} - \text{negative}}$



Non-Negative = $[0, \infty)$

Question

Find the domain of the following functions

i) $f(x) = \sqrt{9 - x^2}$

ii) $f(x) = \dfrac{x-3}{x+4}$

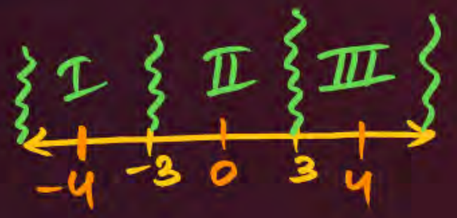
iii) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

iv) $f(x) = \dfrac{1}{\sqrt{x^2+5x+6}}$

$|x| = \sqrt{x^2}$

(i) $f(x) = \sqrt{9-x^2}$

$9 - x^2 \geq 0$



$x^2 \leq 9$

$\sqrt{x^2} \leq \sqrt{9}$

$|x| \leq 3$

$-3 \leq x \leq 3$

$-(x^2-9) \geq 0$

$\Rightarrow x^2 - 9 \leq 0$

$(x-3)(x+3) \leq 0$

$x=3$ $x=-3$

Int.	$(x-3)(x+3)$	Sign
$(-\infty,-3)$	-7×-1	+
$(-3,3)$	-3×3	−
$(3,\infty)$	1×7	+

$-3 \leq x \leq 3$

Question

Find the domain of the following functions

i) $f(x) = \sqrt{9 - x^2}$

ii) $f(x) = \dfrac{x-3}{x+4}$

iii) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

iv) $f(x) = \dfrac{1}{\sqrt{x^2+5x+6}}$ (HW)

(ii) $f(x) = \dfrac{x-3}{x+4}$

$x+4 \neq 0$

$x \neq -4$

$x \in R - \{-4\}$

(iii) $f(x) = \dfrac{2x+3}{x^2-3x+2}$

$x^2-3x+2 \neq 0$

$(x-1)(x-2) \neq 0$

$x \neq 1, x \neq 2$

$x \in R - \{1,2\}$

Question

Find the range of the following functions

i) $f(x) = 5 - 4x$

ii) $f(x) = x^2 + 4$

Pending

iii) $f(x) = x^3 + 1$

iv) $f(x) = x^2 + 4x + 5$

Homework

Find the domain of the function $f(x) = \sqrt{x^2 + 3x + 2}$.

A $(-\infty, -2) \cup (-1, \infty)$

B $(-\infty, -2] \cup [-1, \infty)$

C $(-2, -1)$

D $[-2, -1]$

Homework

Let $A = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ and let $f: A \to N$ be defined by $f(x) =$ the highest prime factor of x. How many elements are there in the range of f?

A 4

B 5

C 6

D 7

THANK
YOU